A PROCEDURE FOR CALCULATING MATRIX HEAT EXCHANGERS FORMED FROM PERFORATED PLATES

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A procedure is proposed for calculating matrix heat exchanger–recuperators, and it is shown that in the general case it is possible to adequately tabulate the efficiency as a function of the length of the apparatus for limited ranges of the important parameters.

Compact matrix recuperator–heat exchangers (CMR's) are widely used in refrigerator engineering [1-3]. Here we will consider one of the types of this equipment which is constructed from thin, perforated metallic plates (thickness δ, with numerous openings), between which there are spacers which are practically non-heat conducting. Narrow rectangular windows are cut in the spacers such that in an assembled group a series of alternating plane-parallel channels (widths $2R_1$ and $2R_2$) is formed, with the "cold" and "hot" gases moving countercurrently to each other in adjacent channels. If the number of channels is sufficiently large, then neglecting end effects, it is possible to calculate such a heat exchanger by considering the heat exchange in an elementary cell consisting of two adjacent half-channels (widths R_1 and R_2) separated by a heat-conducting membrane of width 2b.

Figure 1A shows a sketch of a CMR. A rectangular system of x–y coordinates is selected for any elementary cell. The x axis is taken in the direction of motion of the "hot" gas (x = 0 represents its entry into the equipment), and the y axis is taken in the transverse direction (with y = 0 at the center of the dividing membrane; y = R_1 and y = R_2 are then respectively in the centers of the "hot" and the "cold" channels). In this system of coordinates the equations for the transfer of heat in the channel can be written in the form [4]

$$(\rho_1 u_1 c_1)\frac{\partial t_1'}{\partial x} = \lambda_1 \frac{\partial^2 t_1'}{\partial y^2}, \quad -(\rho_2 u_2 c_2)\frac{\partial t_2'}{\partial x} = \lambda_2 \frac{\partial^2 t_2'}{\partial y^2}. \tag{1}$$

The conditions interrelating the fluxes on both sides of the dividing membrane are of the form

$$\lambda_1\left(\frac{\partial t_1'}{\partial y}\right)\Big|_{y=b} = \lambda_2\left(\frac{\partial t_2'}{\partial y}\right)\Big|_{y=-b}, \quad \lambda_1\left(\frac{\partial t_1'}{\partial y}\right)\Big|_{y=b} = \lambda_T\left(\frac{t_{1s}' - t_{2s}'}{2b}\right). \tag{2}$$

The second equation in (2) indicates that the transfer of heat within the dividing membrane in the x direction is not taken into account, and as a result, the temperature distribution in it along the y axis is linear. The following symmetry conditions must be satisfied at the centers of the two channels:

$$\left(\frac{\partial t_1'}{\partial y}\right)\Big|_{y=R_1} = \left(\frac{\partial t_2'}{\partial y}\right)\Big|_{y=-R_2} = 0.$$

The initial conditions for the inlet temperatures are specified at opposite ends of the heat exchanger

$$t_1'(0,\ y) = t_{10}', \quad t_2'(l,\ y) = t_{20}'.$$

Below use is made of the dimensionless temperatures t_1 and t_2 and the dimensionless coordinates x_1, x_2, y_1, and y_2, defined by the following equations:

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$$t_1' = t_{20} + (t_{10}' - t_{20}')\, t_1, \quad t_2' = t_{20}' + (t_{10}' - t_{20}')\, t_2,$$

$$x = \left(\frac{\rho_1 u_1 c_1 R_1^2}{\lambda_1} \right) x_1 = \left(\frac{\rho_2 u_2 c_2 R_2^2}{\lambda_2} \right) x_2,$$

$$y_1 = (R_1 + b - y)/R_1, \quad y_2 = (R_2 + b + y)/R_2.$$

The coordinates y_1 and y_2 are shown in Fig. 1A. The dimensionless statement of the problem for determining the functions t_1 and t_2 can be rewritten in the form

$$\frac{\partial t_1}{\partial x_1} = \frac{\partial^2 t_1}{\partial y_1^2}, \quad (-1/\beta^2)\frac{\partial t_2}{\partial x_2} = \frac{\partial t_2}{\partial y_2^2}, \tag{3}$$

$$\frac{\partial t_1}{\partial y_1} = (-\varepsilon\beta^2)\frac{\partial t_2}{\partial y_2}, \quad \frac{\partial t_1}{\partial y_1} = -\varkappa(t_{1s} - t_{2s}) \text{ at } y_1 = y_2 = 1,$$

$$\frac{\partial t_1}{\partial y_1} = \frac{\partial t_2}{\partial y_2} = 0 \text{ at } y_1 = y_2 = 0, \tag{4}$$

$$t_1(0, y_1) = 1, \quad t_2(l_1, y_2) = 0.$$

It is easy to see that in the general case there are four independent dimensionless parameters, ε, β, x_2, and κ. On the basis of a theoretical investigation of the problem expressed by equations (3) and (4) which was carried out in [4] it has been shown that in order for it to be possible to tabulate the solution for arbitrary values of the physicochemical and geometrical quantities, it is necessary to introduce in addition to the parameters enumerated above also their linearly independent combinations, namely, two hydrodynamic parameters $X = \log\varepsilon$ and $Y = -\log(\varepsilon\beta)$ which involve the loadings with respect to the two phases, a dimensionless length of the equipment Z, and a parameter which takes into account the conductivity of the wall, P. Following reference [5], we will introduce for reasons of clarity the rectangular planar system of coordinates X–Y, as shown in Fig. 1B. The families of functions I(P, Z) and Σ(P, Z), respectively, will be considered at the points in this plane, depending upon whether they fall to the right ($X \geq 0$) or to the left ($X < 0$) of the midpoint. This is connected with the fact that when $Z \to \infty$, then $I \to 1$ if $X \geq 0$, while on the other hand, $\Sigma \to 1$ if $X < 0$. The values of the parameters Z and P in the left and right havles of the X–Y plane are defined differently [4]:

$$P_+ = (1 + 1/\varepsilon\beta)\sqrt{\varkappa}, \quad Z_+ = l_1/(1 + 1/\varepsilon\beta)^2 \text{ for } X \geqslant 0,$$
$$P_- = (1 + \varepsilon\beta)\sqrt{\varkappa}, \quad Z_- = l_2/(1 + \varepsilon\beta)^2 \text{ for } X < 0. \tag{5}$$

It should be noted that the dimensionless length Z was introduced earlier in a similar way in the investigation of two-phase film-type heat exchangers [5]. It is easy to show that for any values of the parameters X and Y, the solution of the problem given by Eqs. (3) and (4) possesses the property of "symmetry" [4]:

$$I(X, Y, P_+, Z_+) = \Sigma(-X, -Y, P_-, Z_-), \text{ if } P_+ = P_-, Z_+ = Z_-,$$

which makes it possible to reduce by a factor of two the quantity of information needed for the calculation. As a result, in what follows we will confine ourselves to investigating the solution for the right-hand half of the plane only, and the subscripts (+) on the parameters P and Z will be omitted.

The problem (3)-(4) has been solved in [4] by the method of separation of variables. The following formula was obtained for the efficiency:

$$I(X, Y, P, Z) = \sum_r \left[\frac{1 - \exp(-r^2 l_1)}{r^2}\right] A_r - \sum_z \left[\frac{1 - \exp(-z^2 l_2)}{z^2}\right] B_z, \tag{6}$$

where r, z are the discrete roots of the characteristic equation; A_r, B_z are the corresponding coefficients which are determined by solving the system of linear algebraic equations of dimensionality $N = N_r + N_z$, where N_r, N_z are the total number of roots r and z.

All the calculations were carried out for $N_r = N_z = 20$. Checking of the calculations for $N_r = 30$, 40, and 60 led to practically no change in the results for the length being considered. By formally investigating the forms of the problem (3)-(4) for $\varkappa = \infty$ and $\varkappa = 0$, it

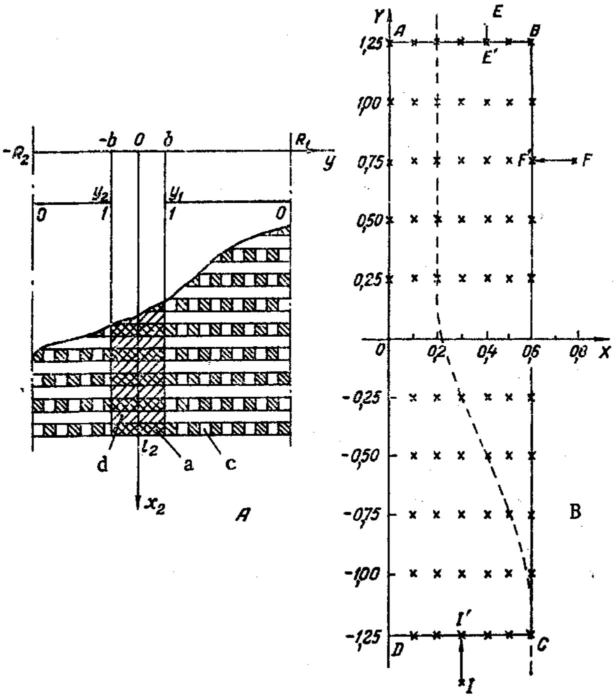

Fig. 1

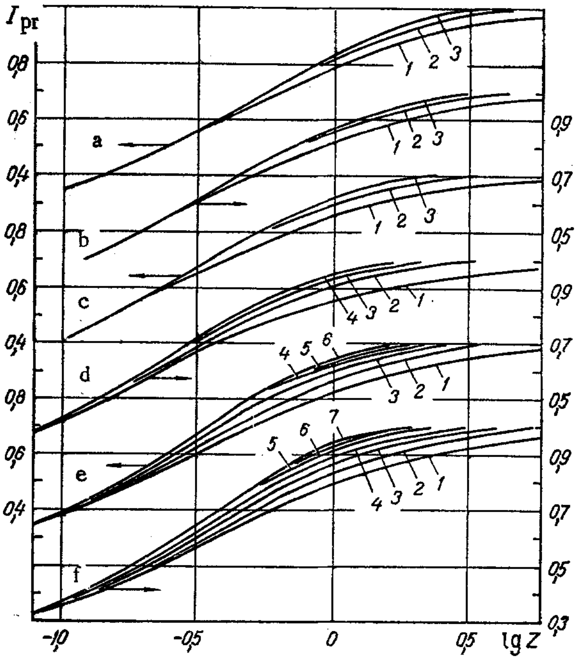

Fig. 2

Fig. 1. Sketch of heat exchanger (A) [a) perforated plate; c) perforation; d) spacer], and plane of the hydrodynamic variables (B) [the dashed curve represents the relationship $X_\infty(Y)$].

Fig. 2. Efficiency as a function of the length for infinite conductivity of the wall; a) Y = 1.25; b) 0.75; c) 0.25; d) −0.25; e) −0.75; f) −1.25; 1) X = 0; 2) 0.1; 3) 0.2; 4) 0.3; 5) 0.4; 6) 0.5; 7) 0.6.

was possible to obtain two limiting solutions. In the first case ($\varkappa = \infty$), as can be easily seen from the boundary conditions (4), the equality $t_{1S} = t_{2S}$ is satisfied everywhere within the equipment, and as a result, the temperature distributions in the channels are the same as in the problem of two-phase heat transfer with an infinitely thin dividing membrane (b = 0). The solution of the latter problem was found earlier by numerical means [5] using the method of iteration, where the efficiency as a function of the dimensionless length, or $I_{pr}(X, Y, Z)$, was calculated at a number of discrete points in the X–Y plane. Since this solution is of considerable importance in itself, and also bearing in mind its importance for the discussion below, $I_{pr}(Z)$ was also calculated in the present work by the method of separation of variables, i.e., by Eq. (6). It should be noted that compared to the numerical method this latter method is almost instantaneous and is also more accurate. The results are shown in Fig. 2. In the general case $I_{pr}(Z)$ depends on both the parameters X and Y. It can easily be seen that at any level (i.e., at a fixed value of Y) the function $I_{pr}(Z)$ increases with increase of the parameter X and tends towards the limiting curve $i_{pr}(\infty, Y, Z)$. In practice the approximate equation

$$I_{pr}(X, Y, Z) \simeq I_{pr}(\infty, Y, Z) \tag{7}$$

can be used starting at some value $X = X_\infty$, the value of which depends on the level and also on the degree of approximation towards the limit. Subsequently, the equations similar to (7) will be regarded as accurate if the relative error between the functions expressed by the left-hand and right-hand parts does not exceed 2% for any value of Z considered. The curve $X_\infty(Y)$ is shown by the dashed line in Fig. 1B. Thus, everywhere in the X–Y plane the curve $X_\infty(Y)$ for the efficiency $I_{pr}(Z)$ depends only on the parameter Y. On the other hand, at large and small values of Y this function depends practically only on X, and is given by the corresponding limiting relationship:

$$I_{pr}(X, Y, Z) \simeq I_{pr}(X, \pm\infty, Z). \tag{8}$$

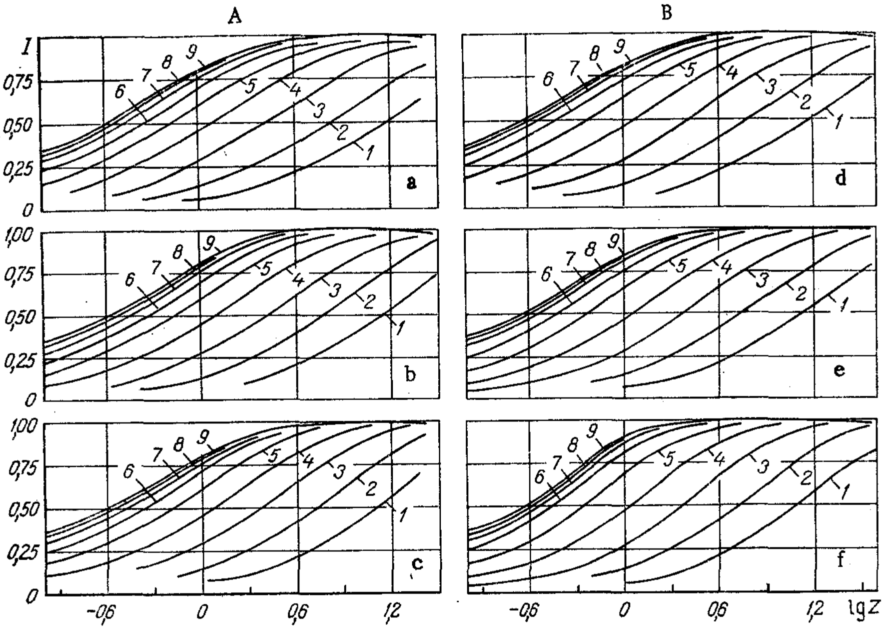

Fig. 3. Efficiencies as a function of length at the level Y = 1.25 (A) and Y = −1.25 (B): a, d) X = 0.1; b, e) X = 0.2; c, f) X = 0.4; 1) log P = −0.6; 2) log P = −0.4; 3) log P = −0.2; 4) log P = 0; 5) log P = 0.2; 6) log P = 0.4; 7) log P = 0.6; 8) log P = 0.8; 9) log P = 1.0.

The calculations showed that to the accuracy assumed above, Eq. (8) is satisfied in the zones $Y \geq 1.25$ and $Y \leq -1.25$, respectively. The physical reason for this behavior of the function $I_{pr}(Z)$ was established in [5]. The solution for $\varkappa = \infty$ corresponds to the case in which the thermal resistance of the wall is equal to zero, and its conductivity is infinite.

In the other limiting case ($\varkappa = 0$), the equations $t_1 = t_{1s}$, $t_2 = t_{2s}$ are satisfied almost everywhere within the CMR. By integrating the transfer Eq. (3) with respect to y_1 and y_2 from zero to unity, a system of ordinary first-order equations

$$\varkappa (t_1 - t_2) = -\frac{\partial t_1}{\partial x_1}, \quad t_1 - t_{1l} = \varepsilon t_2,$$

is obtained instead of the partial differential equations, where the second relationship represents the heat-balance condition. As a result of solving this system of equations with the boundary conditions $t_1(0) = 1$, the following analytical expression is obtained for the efficiency:

$$\begin{aligned} I_0(X,\ P^2Z) &= \{1 - \exp[-(1 - 1/\varepsilon)\, l_1 \varkappa]\}/\{1 - \exp[-(1 - 1/\varepsilon)\, l_1 \varkappa]/\varepsilon\} = \\ &= \{1 - \exp[-(1 - 1/\varepsilon)\, P^2 Z]\}/\{1 - \exp[-(1 - 1/\varepsilon)\, P^2 Z]/\varepsilon\}. \end{aligned} \tag{9}$$

If we restrict ourselves to a 2% accuracy, simple calculations show that the approximation

$$I_0(X,\ P^2Z) \simeq I_0(\infty,\ P^2Z) = 1 - \exp(-P^2Z) \tag{10}$$

is satisfied when $X \geq 0.6$. If it is taken into account that $I_0(P^2Z)$ depends only on X, then Eq. (10) means that for $\varkappa = 0$ and with the accuracy assumed above, the vertical line X = 0.6 can serve as the boundary of the zone in which $I_0(P^2Z)$ changes. From its physical meaning, Eq. (9) corresponds to the case in which the thermal resistance of both channels in the transverse direction can be neglected compared with the corresponding resistance of the membrane. In other words, the overall conductivity of the channel tends to infinity.

By using the limiting solutions I_{pr} and I_0 it is possible to look into the physical significance of the parameter P which is defined by Eq. (5). In reality, when the thermal resistance of the separating membrane can be neglected the characteristic length of the CMR (i.e., the length at which $I \doteq 1$) is of the order of $l_1 \approx (1 + 1/\varepsilon\beta)^2$ (Fig. 2). If it is possible to ignore the thermal resistance of the channel, then $l_1 \approx 1/\varkappa$, as follows from Eq. (9). Since the parameter P^2 is equal to the ratio of these characteristic lengths, then qualitatively the heat transfer in the CMR can be regarded as the result of two competing processes. For large values of P the limiting process is heat transfer in the channels, and in the case $I \simeq I_{pr}$; at small values of P the limiting process is the transfer of heat through the dividing wall, and here $I \simeq I_0$. Consequently, the physical significance of the parameter P is a relative conductivity of the dividing membrane, the range of variation of which can be taken as finite everywhere within the X–Y plane.

The solution can be represented graphically in terms of the selected variables X, Y, and P. In fact, the curves belonging to the family of functions I(P, Z), where P is taken as a parameter, fall in the general case between two limiting relationships: $I_{pr}(Z)$ and $I_0(P^2 Z)$ (for sufficiently small values of P). Figure 3A shows the results of calculations of the efficiencies at the level Y = 1.25 for various values of X. The parameter log P assumes a number of discrete values in the range [−0.6, 1] in steps of 0.2. Outside of this range, the approximation (7) is valid everywhere with the accuracy assumed above (2%) when the inequality $\log P \geq 1$ is satisfied, while the solution coincides with the analytical Eq. (9) when $\log P \leq -0.6$.

If the limiting solutions I_{pr} and I_0 and the positions of the boundary curves $X_\infty(Y)$ are analyzed at large and small values of the conductivity, then, bearing in mind the monotonic nature of the effect of P, the conclusion is reached that in the general case it is sufficient in calculating I(P, Z) to restrict our attention to the finite region of the X–Y plane given by $0 \leq X \leq 0.6$, $-1.25 \leq Y \leq 1.25$, i.e., to the rectangle ABCD in Fig. 1B. The points corresponding to this zone at which calculations have been carried out are shown by crosses in Fig. 1B. For example, Fig. 3B shows the calculations for the points falling on the lower side (CD) of this rectangle. As can be seen from the graphs in Fig. 3 which were obtained for the extreme values of Y (Y = ±1.25), the values of log P and the steps by which it varies can be taken to be the same in all the cases. The calculations showed that this same behavior was observed everywhere within the rectangle ABCD. The latter is very important for obtaining I(P, Z) at any internal point in ABCD (for any fixed value of P), since for constant values of the range [−0.6, 1] it is possible to easily extrapolate the solutions obtained for discrete points at any values of X, Y corresponding to ABCD. If the points fall outside the limits of the rectangle ABCD (for example, the points E, F, J in Fig. 1B), then it is sufficient to drop perpendicularly from the point to the nearest side of the rectangle ABCD. It is obvious that the following relationship exists between the efficiencies at the given points and those at the points of intersection of the perpendiculars with the sides of the rectangle ABCD (E', F', J' in Fig. 1B):

$$I(P, Z) \simeq I(P', Z') \quad \text{for} \quad P = P', \; Z = Z',$$

where P', Z' are the parameters calculated from Eq. (5) for the boundary points (E', F', J').

Thus, as the basis for the calculation of CMR's with selected steps along the axes of $\Delta X = 0.1$ and $\Delta Y = 0.25$ for arbitrary conditions, i.e., any permissible values of the parameters X, Y, and P ($-\infty \leq X, Y, \log P \leq +\infty$), 77 graphs have been set up for the families of functions I(P, Z) tabulated in the following restricted ranges of variation of these parameters: $0 \leq X \leq 0.6$, $-1.25 \leq Y \leq 1.25$, $-0.6 \leq \log P \leq 1$.

It has not been possible to give all of the graphs in the present paper. As examples, six of them are shown in Fig. 3. It is obvious that the quantity of information which is needed depends greatly on the accuracy which is required in the calculations, which is related to the choice of the steps along the X and Y axes. Here this choice was governed by the requirement that the error in the calculation of the length Z should not exceed 2-4%, which is acceptable for practical calculations. The calculations were carried out on a BÉSM-6 computer. Subsequently, these calculations served as the basis of an "economic" procedure for calculating CMR's in the use of which relationships are required only for the limiting values of the parameters $X = +\infty$, $Y = \pm\infty$.

NOTATION

c_i, Heat capacity at constant pressure, J/deg; u_i mass-average velocity, m/sec; $I = 1 - \bar{t}_1 l$, efficiency, or dimensionless heat flux for "hot" channel; $\Sigma = \bar{t}_2 l$, dimensionless heat flux for "cold" channel; l, length of heat exchanger, m; $l_i = l/R_i Pe_i$, dimensionless length of CMR; t'_i, temperature, °; t'_{i0}, constant temperature at the entry of the gas into the heat exchanger: $\bar{t}_i = \int_0^1 t_i dy_i$, dimensionless temperatures; ρ_i, density, kg/m³; $\beta^2 = R_1 Pe_1/R_2 Pe_2$, dimensionless parameter; $\varepsilon = R_2\rho_2u_2c_2/R_1\rho_1u_1c_1$, dimensionless parameter; $\varkappa = (\lambda_T/\lambda_1)(R_1/2b)$, dimensionless parameter; $Pe_i = \rho_iu_ic_iR_i/\lambda_i$, thermal Péclet number. Subscripts: s, surface of separating membrane; i = 1, 2, "hot" and "cold" channels, respectively; l, outlet of gas; T, separating membrane.

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DETERMINING THE THERMOPHYSICAL CHARACTERISTICS OF MATERIALS ON A MODEL OF A SEMIINFINITE BODY WITH HEAT SUPPLIED BY MEANS OF A THIN ANNULAR HEATER

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Methods of complex calculation of the thermophysical characteristics of materials without destroying their integrity are proposed, on the basis of a model of a semiinfinite body on heating with a rectangular heat pulse through a specified annular region.

The problem posed here is the complex determination of the thermophysical characteristics of materials (without loss of their integrity) using a model of a semiinfimite (in thermal terms) body with a pulsed heat supply to its surface. A short heat pulse (of rectangular form) acts in a limited annular region, and the excess temperature $T_i(r, z, \tau)$ is measured at a point coinciding with the center of the annular heat source.

To solve this problem the two-dimensional nonsteady temperature field $T_i(r, z, \tau)$ in cylindrical coordinates must be determined as a function of the heat-flux density $q(\tau)$ acting in a finite annular region $R_1 \le r \le R_2$, where R_1 and R_2 are the radii of the annular heater at the surface of the semiinfinite body when $z = 0$; $R_2 > R_1$. In the regions of variation $r < R_1$ and $R_2 < r < \infty$ on the surface, there is assumed to be no temperature gradient along the normal to the boundary of the body. The initial temperature distribution is assumed to be constant: T_0 = const. The origin of the cylindrical coordinates ($r = z = 0$) is chosen at the center of the annular heater.

The mathematical problem is formulated as a system of three differential heat-conduction equations of the form

$$\frac{\partial^2 T_i(r, z, \tau)}{\partial r^2} + \frac{1}{r}\frac{\partial T_i(r, z, \tau)}{\partial r} + \frac{\partial^2 T_i(r, z, \tau)}{\partial z^2} = \frac{1}{a}\frac{\partial T_i(r, z, \tau)}{\partial \tau} \quad (i = 1, 2, 3). \tag{1}$$

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